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POSSIBLE PROTON-ALPHA SCATTERING EXPERIMENTS AT 40 MEV

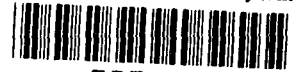
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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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EXPERIMENTS AT 40 MEV *

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SUMMARY

Two qualitatively different phase-shift analyses of the 40-Mev proton-alpha elastic-scattering and polarization data have appeared in the literature. The question arises whether further experimental data can decide in favor of one or the other of these analyses. Two classes of experiments, measurement of the triple-scattering parameters R and A and measurement of the polarization P in the region of the Coulomb interference scattering cross section minimum, have been examined to determine if these provide additional information to resolve the ambiguity. It is concluded that a measurement of the polarization near the Coulomb interference minimum would serve to establish, on a purely experimental basis, the complete scattering amplitude in the forward direction. This information would remove the ambiguity in the phase-shift analysis of these data. No further information could be gained from triple scattering experiments unless these experiments were of very high precision.

INTRODUCTION

Recently, several different phase-shift analyses (refs. 1 and 2) of the 40-Mev proton-alpha scattering (ref. 3) and polarization (ref. 4) data appeared in the literature. The present analysis has also produced a number of sets of phase shifts that fit these data equally well. It is thus self-evident that there does not exist a unique phase-shift analysis. Furthermore, a brief glance at the scattering amplitudes themselves reveals that this lack of uniqueness does not result from any inherent ambiguity in the expression of the scattering amplitude in terms of phase shifts. The scattering amplitudes themselves differ appreciably.

The question then arises whether this ambiguity can be resolved with the aid of further experimental data. In reference 5, it is shown that, in prin-

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ciple, measurement of the rotation of polarization can provide additional information. Such experiments are, of course, exceedingly difficult. Therefore, it is desirable to examine in advance what might be learned from such an experiment in the case of the scattering of protons by alpha particles at about 40 Mev.

SYMBOLS

A	triple-scattering parameter, dimensionless
f_{coul}	Coulomb scattering amplitude in absence of nuclear forces, fm
G_0	limiting value as $\theta \rightarrow 0$ of magnitude of "nuclear with Coulomb" spin-independent scattering amplitude, fm
g	complex spin-independent scattering amplitude, fm
H_0	limiting value as $\theta \rightarrow 0$ of magnitude of coefficient of θ in spin-dependent scattering amplitude, fm
h	complex spin-dependent scattering amplitude, fm
Im	imaginary part
J	total angular momentum quantum number, dimensionless
k	wave propagation number, fm^{-1}
\vec{k}	wave propagation vector, fm^{-1}
l	orbital angular momentum quantum number, dimensionless
P	polarization, dimensionless
P_l	Legendre polynomial of order l
P_l'	derivative of P_l with respect to θ
R, R'	triple-scattering parameters, dimensionless
Re	real part
\vec{S}	vector normal to second scattered beam \vec{k}_f , dimensionless
β	rotation angle, deg or radians
Γ	phase angle of spin-independent scattering amplitude, deg or radians
γ	limiting value as $\theta \rightarrow 0$ of phase angle of "nuclear with Coulomb" spin-independent scattering amplitude, deg or radians

δ_l^+, δ_l^-	nuclear phase shifts for $J = l + \frac{1}{2}$ and $J = l - \frac{1}{2}$, respectively, radians
η	standard parameter for strength of Coulomb interaction, dimensionless
θ	center-of-mass scattering angle, deg or radians
θ_{lab}	laboratory scattering angle, deg or radians
λ	limiting value as $\theta \rightarrow 0$ of phase angle of spin-dependent scattering amplitude, deg or radians
$\overline{\langle \vec{\sigma} \rangle}$	statistical average of spin vector, dimensionless
$\sigma(\theta)$	elastic scattering cross section, fm ²
σ_l	pure Coulomb phase shift of order l , dimensionless

Subscripts:

coul	Coulomb
f	beam scattered from second scatterer
i	beam incident on second scatterer
lab	laboratory
nuc	nuclear
1	derived from phase shifts of ref. 1
2	derived from phase shifts of ref. 2

Superscripts:

(*)	complex conjugate
(')	in conjunction with P_l , derivative; in conjunction with R , serves to distinguish two different but related parameters

PROCEDURE

Consider a 100-percent polarized beam of spin $1/2$ particles with the polarization vector normal to the incident beam and in the plane of scattering as shown in figure 1. If a scattering experiment were to be performed with such an initial beam, the polarization of the scattered beam could be measured. In such an experiment the component of polarization of the scattered beam in the plane of scattering and normal to the scattering direction is the rotation of polarization parameter R . The sign of R is purely conventional. The convention used here is

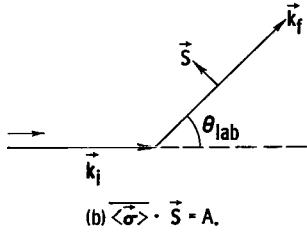
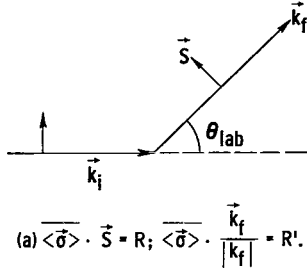


Figure 1. - Triple-scattering experiments. Only scattering from second target is shown. Small arrow on beam incident on second scatterer k_i indicates direction of polarization from first scattering (not shown). Arrow on outgoing beam k_f indicates normal \vec{S} to third scattering plane. Equations for measured component of polarization are given for case in which incident beam is completely polarized.

$$\vec{R} = \overline{\langle \vec{\sigma} \rangle} \cdot \vec{S} \quad (1)$$

where

$$\vec{S} = \frac{(\vec{k}_i \times \vec{k}_f) \times \vec{k}_f}{|(\vec{k}_i \times \vec{k}_f) \times \vec{k}_f|} \quad (2)$$

as shown in figure 1, which is drawn in the manner of reference 5.

A straightforward calculation yields the standard result (ref. 5)

$$R = (1 - P^2)^{1/2} \cos(\beta - \theta_{lab}) \quad (3)$$

where θ_{lab} is the laboratory scattering angle, and the parameters P and β are given by the relations

$$\left. \begin{aligned} P(\theta) &= \frac{2\text{Re}[g^*(\theta)h(\theta)]}{[|g(\theta)|^2 + |h(\theta)|^2]} \\ \cos \beta &= \frac{|g(\theta)|^2 - |h(\theta)|^2}{[|g(\theta)|^2 + |h(\theta)|^2] [1 - P(\theta)^2]^{1/2}} \\ \sin \beta &= \frac{2\text{Im}[g(\theta)h^*(\theta)]}{[|g(\theta)|^2 + |h(\theta)|^2] [1 - P(\theta)^2]^{1/2}} \end{aligned} \right\} \quad (4)$$

where θ is the center-of-mass scattering angle. The quantities $g(\theta)$ and $h(\theta)$ are the spin-independent and spin-dependent scattering amplitudes, respectively. In terms of phase shifts, $g(\theta)$ and $h(\theta)$ can be expressed as

$$g(\theta) = f_{coul} + \frac{1}{k} \sum_l e^{2i\sigma_l} \left[(l+1)e^{i\delta_l^+} \sin \delta_l^+ + l e^{i\delta_l^-} \sin \delta_l^- \right] P_l(\cos \theta) \quad (5)$$

$$h(\theta) = \frac{1}{2k} \sum_l e^{2i\sigma_l} \left(e^{2i\delta_l^+} - e^{2i\delta_l^-} \right) \sin \theta \frac{dP_l(\cos \theta)}{d(\cos \theta)} \quad (6)$$

where f_{coul} is the Coulomb scattering amplitude in the absence of the nuclear force, σ_l is the Coulomb phase shift, and δ_l^+ and δ_l^- are the nuclear phase shifts for $J = l + \frac{1}{2}$ and $J = l - \frac{1}{2}$, respectively.

The phase shifts generated in the investigation of reference 1 may be substituted into the previous expressions; then R can be calculated and compared with the results of reference 2 and with those of the present analysis. Three sets of phase shifts are presented in table I. The results for β and R are plotted for two phase shifts in figure 2 and for three phase shifts in figure 3.

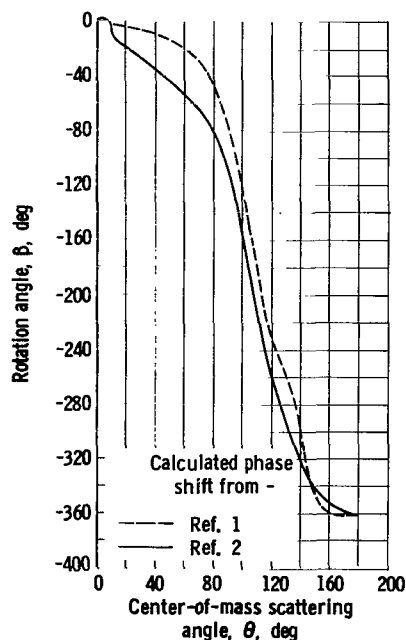


Figure 2. - Rotation angle as function of center-of-mass scattering angle for two sets of phase shifts.

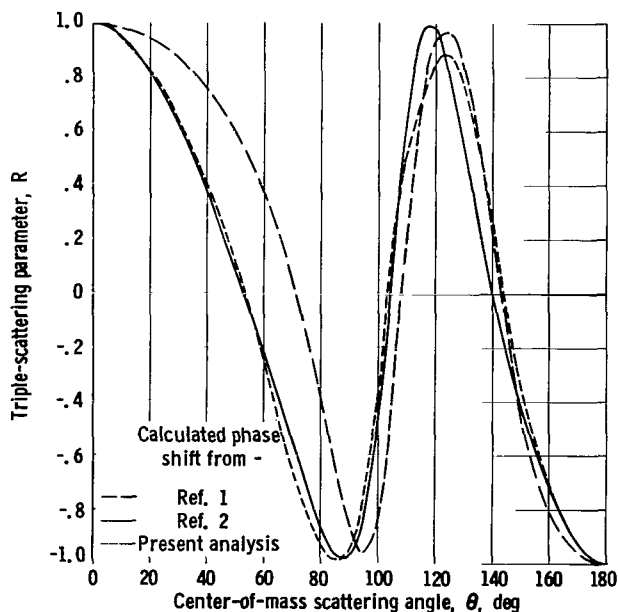


Figure 3. - Rotation parameter as function of center-of-mass scattering angle for three sets of phase shifts.

The triple-scattering experiments are difficult to perform so that attention will be restricted to the forward angles, since the cross section falls very rapidly with increasing angle, as shown in figure 4. Very forward angles are essentially pure Coulomb scattering and hence yield no information. From figure 2, it can be seen that β is nearly a linear function of θ for values of θ from about 10° to 50° . It will be shown that this linearity is theoretically necessary. Hence, any measurement in the linear region is equivalent to any other. This implies that the measurements should be made as far forward as possible in order to maximize the number of events that can be observed. In the rotation experiment, however, this is the region where the value of R is least sensitive to the differences in β . For example, an experiment performed with an accuracy of ± 10 percent in the measurement of R at $\theta = 30^\circ$, could not serve to distinguish between the two phase shifts plotted in figure 3.

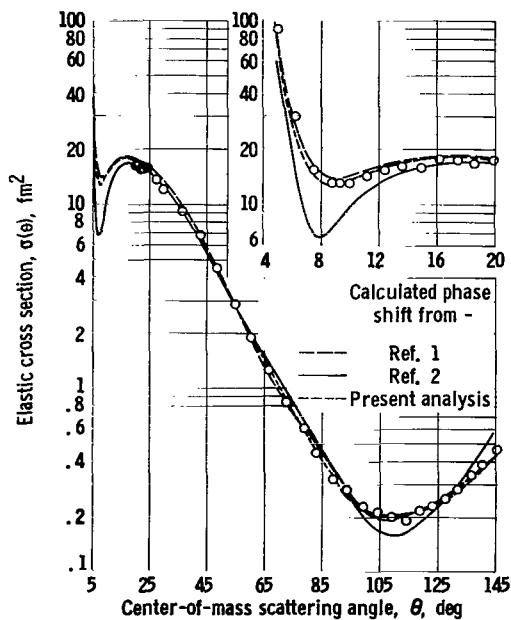


Figure 4. - Elastic cross section as function of center-of-mass scattering angle for three sets of phase shifts compared with data of reference 3. Circles representing data points are larger than error bars assigned in reference 3.

If Coulomb effects are neglected, $g(\theta)$

and $h(\theta)$ may be estimated to first order in θ to be

$$g(\theta) = \frac{1}{k} \sum_l \left[(l+1) e^{i\delta_l^+} \sin \delta_l^+ + l e^{i\delta_l^-} \sin \delta_l^- \right] + O(\theta^2) = g(\theta=0) + O(\theta^2) \quad (7)$$

and

$$h(\theta) = \frac{1}{2k} \sum_l \left(e^{2i\delta_l^+} - e^{2i\delta_l^-} \right) \theta P_l'(\theta=0) + O(\theta^3) = \left(\frac{dh}{d\theta} \right)_{\theta=0} \theta + O(\theta^3) \quad (8)$$

The quantities $g(\theta=0)$ and $\left[\frac{dh(\theta)}{d\theta} \right]_{\theta=0}$ calculated according to equations (7) and (8) are also given in table II where they are listed as $|G_0|e^{i\gamma}$ and $|H_0|e^{i\lambda}$, respectively. When equations (7) and (8) are applied to equation (6) the following form for β is obtained:

$$\beta = \left(\frac{d\beta}{d\theta} \right)_{\theta=0} \theta + O(\theta^3) \quad (9)$$

This accounts for the nearly linear region in figure 2, where, for the two phase shifts plotted, the ratio of the calculated values of β is

$$\frac{\beta_2}{\beta_1} = \frac{-0.81}{-0.15} \theta = 5.4 \quad (10)$$

From equation (3), however, it can be seen that this large difference is somewhat masked by the fact that the rotation of polarization depends on β through the factor $\cos(\beta - \theta_{lab})$.

Other triple-scattering experiments are possible. All these experiments are not useful for obtaining additional information in the present case where spin 1/2 particles are scattered from spin zero targets. The so-called depolarization parameter is always unity and is, hence, of no interest. The triple-scattering parameters R' and A depend on $P(\theta)$, $\beta(\theta)$, and θ_{lab} according to the relations

$$-R' = A = [1 - P(\theta)^2]^{1/2} \sin(\beta - \theta_{lab}) \quad (11)$$

These experiments are inherently more difficult to perform than the R experiment, because they involve the rotation of the spin vector by 90° in a magnetic field.

The meaning of the quantities R' and A is discussed in reference 5 and is illustrated in figure 1. The values of A calculated for the various sets of phase shifts under discussion are illustrated in the two most different instances in figure 5.

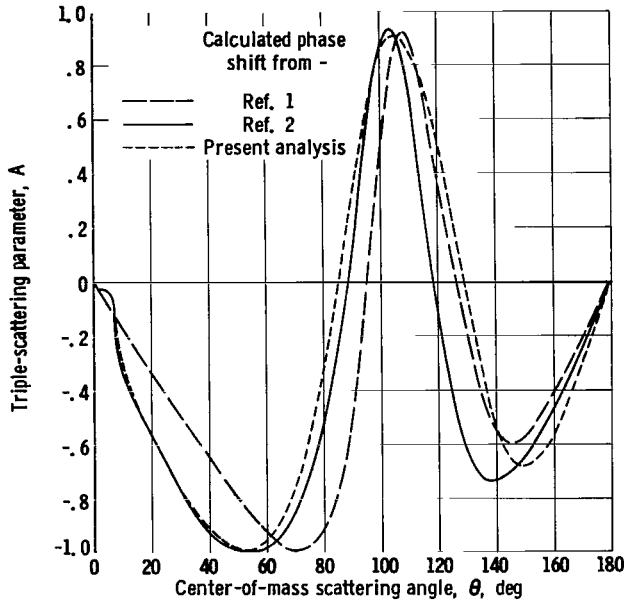


Figure 5. - Triple-scattering parameter as function of center-of-mass scattering angle for three sets of phase shifts.

Both R and R' depend on $\beta(\theta)$ in the combination $\beta - \theta_{lab}$. For fairly small angles,

$\beta(\theta) \approx \frac{d\beta}{d\theta}(\theta=0)\theta$ and $\theta_{lab} \approx 0.8\theta$, so that $\beta(\theta) - \theta_{lab} \approx \left[\frac{d\beta(0)}{d\theta} - 0.8\right]\theta$. For the two cases chosen as illustrations, the values of $d\beta(0)/d\theta$ appear in equation (10). For these values

$$\beta_2(\theta) - \theta_{lab} \approx -1.61\theta \quad (12)$$

and

$$\beta_1(\theta) - \theta_{lab} \approx -0.95\theta \quad (13)$$

so that

$$\frac{R_2}{R_1} \approx 1 - \frac{1}{2}(1.61^2 - 0.95^2)\theta^2 =$$

$$1 - 0.84\theta^2 \quad (14)$$

and

$$\frac{R_2}{R_1} \approx \frac{1.61}{0.95} = 1.7 \quad (15)$$

Neither difference could be seen for angles less than 30° in an experiment performed with an accuracy of ± 10 percent.

The information that could have been gained by high-precision triple-scattering experiments in the forward cone can in actual fact be attained more readily through a double-scattering experiment in the region of the cross section dip caused by the interference between the Coulomb and the nuclear scattering amplitudes. In a forward cone, which includes the Coulomb interference region, the spin-independent amplitude is nearly constant, while the spin-dependent amplitude is nearly a constant times θ . If the Coulomb terms are included, equations (7) and (8) become

$$g(\theta) = g_{coul}(\theta) + g_{nuc}(\theta) \approx g_{coul}(\theta) + g_{nuc}(0)$$

$$= \frac{\eta}{2k \sin^2(\theta/2)} e^{2i[\sigma_0 - \eta \ln(\sin \theta/2) + (\pi/2)]} + \frac{1}{k} \sum_l e^{2i\sigma_l} [(l+1)e^{i\delta_l^+} \sin \delta_l^+ + le^{i\delta_l^-} \sin \delta_l^-]$$

$$\equiv g_{coul}(\theta) + |G_0| e^{i\gamma} \equiv |g(\theta)| e^{i\Gamma(\theta)} \quad (16)$$

and

$$\begin{aligned} h(\theta) &\approx \left(\frac{dh}{d\theta} \right)_{\theta=0} \theta = \frac{1}{2k} \sum_l e^{2i\sigma_l} \left(e^{2i\delta_l^+} - e^{2i\delta_l^-} \right) P_l^1(\theta=0) \theta \\ &\equiv |H_0| e^{i\lambda} \theta \end{aligned} \quad (17)$$

To lowest order in θ the elastic cross section $\sigma(\theta)$ is given by

$$\sigma(\theta) \approx \sigma_{\text{coul}} + 2|G_0| \text{Re} \left[e^{-i\gamma} g_{\text{coul}}(\theta) \right] + |G_0|^2 \quad (18)$$

Examination of the experimentally determined angular distribution in the vicinity of the Coulomb interference minimum can now serve to identify $|G_0|$ and γ . When the data at 6° and at 9° are used, the results are

$$|G_0| = 4.9 \quad (19)$$

and

$$\gamma = 59^\circ \quad (20)$$

The calculated values for $|G_0|$ and γ for the scattering amplitudes in the present discussion appear in table II.

If $\left[\frac{\partial h(\theta)}{\partial \theta} \right]_{\theta=0}$ can be determined in a similar fashion, all the information necessary to give the complete scattering amplitude in the forward cone will be available. Polarization data in the Coulomb interference region can give this information. For small angles the polarization is

$$P(\theta) = \frac{2\text{Re}[g^*(\theta)h(\theta)]}{|g(\theta)|^2 + |h(\theta)|^2} \approx \frac{2|H_0| \cos[\Gamma(\theta) - \lambda]}{\sqrt{\sigma(\theta)}} \theta \quad (21)$$

where $|g(\theta)|$ and $\Gamma(\theta)$ are as defined in equations (16), (19), and (20). An experimental knowledge of the polarization at two angles in the Coulomb interference region will then serve to determine $|H_0|$ and λ .

The polarization for the phase shifts under discussion is shown in figure 6. The polarizations predicted by the phase shifts of reference 2 and the present analysis differ significantly from the polarizations predicted by the phase shifts of reference 1 in the forward cone.

Suppose that polarization data at 8° and 16° were available to compare with the predictions of the phase-shift analyses. The values of $\Gamma(\theta)$ are determined by the cross section data to be

$$\Gamma(\theta = 8^\circ) = 125^\circ \quad (22)$$

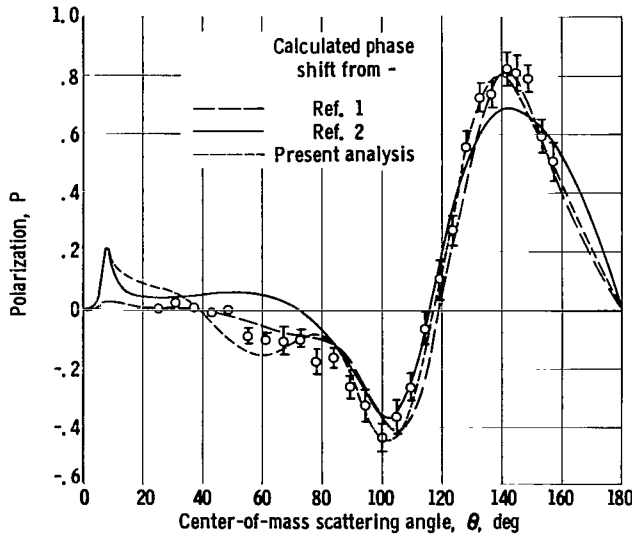


Figure 6. - Polarization as function of center-of-mass scattering angle for three sets of phase shifts compared with data of reference 4.

The values of $|H_0|$ and λ for the three illustrative phase shifts under discussion appear in table II. From that table it is apparent that the principal qualitative difference between the results of references 1 and 2 is that the value of $|H_0|$ from reference 1 is small. Thus, no matter what the phase of the spin-dependent forward scattering amplitude, a small polarization may be expected everywhere in the forward cone.

On the other hand, the value of $|H_0|$ from reference 2 is large. The existing polarization data in the forward cone indicate that the polarization is small beyond the Coulomb interference region. If $|H_0|$ is large, this small polarization can come about only if the spin-dependent amplitude is approximately $\pi/2$ out of phase with the spin-independent amplitude. This is, in fact, true for all cases under consideration. From table II it is apparent that $0 < (\gamma - \lambda + \pi/2) < 0.2$ radians for all cases under discussion. If $|H_0|$ is sufficiently large, however, substantial polarization in the Coulomb interference region may be expected. At the Coulomb interference minimum ($\theta = 8^\circ$) the phase of the spin-independent scattering amplitude is 125° . If the phase λ of the spin-dependent amplitude were such that $\Gamma - \lambda + \pi/2$ turned out to be near $\pi/2$ in the Coulomb interference region, a large polarization could result. For $|H_0|$ of approximately 2.5 femtometers, from equation (26), a maximum polarization of 18 percent at $\theta = 8^\circ$ could result for $\Gamma = \lambda$. For $|H_0|$ of approximately 0.4 femtometer, there can be a maximum polarization of no more than 3 percent.

CONCLUSION

In view of the previous arguments, it may be concluded that a measurement of the polarization in the vicinity of the Coulomb interference minimum in

and

$$\Gamma(\theta = 16^\circ) = 72^\circ \quad (23)$$

The values of $\sqrt{\sigma(\theta)} = |g(\theta)|$, taken from reference 3, are

$$g(\theta = 8^\circ) = 3.8 \text{ fm} \quad (24)$$

$$g(\theta = 16^\circ) = 4.2 \text{ fm} \quad (25)$$

which give

$$P(\theta = 8^\circ) = 0.073 |H_0|$$

$$\sin\left(125^\circ - \lambda + \frac{\pi}{2}\right) \quad (26)$$

and

$$P(\theta = 16^\circ) = 0.132 |H_0|$$

$$\sin\left(72^\circ - \lambda + \frac{\pi}{2}\right) \quad (27)$$

proton-alpha scattering at 40 Mev would serve to establish, on a purely experimental basis, the complete scattering amplitude in the forward direction. Furthermore, this additional information would serve to remove the ambiguity in the complete phase-shift analysis of these data. No further information could be gained through the more difficult triple-scattering experiments unless these were of high precision.

Lewis Research Center

National Aeronautics and Space Administration

Cleveland, Ohio, March 27, 1964

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TABLE I. - PHASE SHIFTS THAT FIT PROTON-ALPHA ELASTIC
SCATTERING AND POLARIZATION DATA AT
APPROXIMATELY 40 MEV

Phase shift from-	Predicted reaction cross section, fermis ²	Angular momentum quantum number, l	Nuclear phase shift			
			Real		Imaginary	
			δ_l^+	δ_l^-	δ_l^+	δ_l^-
Ref. 1	11.6	0	1.289	-----	0.0877	-----
		1	1.318	0.8835	.160	0.023
		2	.397	.228	.0554	.140
		3	.209	.186	.0599	.065
Ref. 2	0	0	0.987	-----	-----	-----
		1	1.144	0.572	-----	-----
		2	.432	.156	-----	-----
		3	.233	.115	-----	-----
		4	.075	.001	-----	-----
Present analysis	10.3	0	1.161	-----	0.091	-----
		1	1.299	0.5897	.029	0.026
		2	.416	.1142	.143	.068
		3	.224	.1237	.082	.030
		4	.062	.005	-----	-----
		5	.026	.016	-----	-----

TABLE II. - CALCULATED VALUES OF SCATTERING PARAMETERS
AS DEFINED IN EQUATIONS (16) AND (17)

	Scattering parameter			
	From ref. 1 (a)	From ref. 2 (a)	From present analysis (a)	From cross section data of ref. 3 (b)
$ G_0 $, fm	5.46	5.54	5.48	4.9
γ , deg	56	40.2	53.2	59
$ H_0 $, fm	0.42	2.24	2.64	---
λ , deg	144.3	129.3	132.6	---
$\gamma - \lambda + \frac{\pi}{2}$, deg	1.7	0.9	10.6	---
$g(8^\circ)$, fm	3.84	2.57	3.62	3.8
$\Gamma(8^\circ)$, deg	117.2	104.3	115.6	125
$\Gamma(8) - \lambda + \frac{\pi}{2}$, deg	62.9	65.0	73.0	---
$P(8^\circ)$	0.03	0.21	0.18	---
$g(16^\circ)$, fm	4.26	4.00	4.15	4.2
$\Gamma(16^\circ)$, deg	69.8	51.8	67.6	72
$\Gamma(16) - \lambda + \frac{\pi}{2}$, deg	15.7	12.5	25	---
$P(16^\circ)$	0.01	0.05	0.11	---

^aCalculated by using approximations to lowest order in scattering angle; values of elastic cross section and polarization used were computed exactly from phase shifts of table I.

^bComputed from cross sections at 6° and 9° .

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